Using the Binomial Transformer to Approximate the Q Distribution for Maximally Flat Quarter-Wavelength-Coupled Filters

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***Abstract*— With the binomial transformer as a basis, a closed-form expression is derived for the Q distribution used to design maximally flat quarter-wavelength-coupled transmission-line filters. The derived expression is shown to be valid for filters with small values of total Q. Also, an existing Q-distribution expression derived from the lumped-element prototype (LEP) circuit is discussed, and is shown to be valid for filters with large values of total Q. By combining the derived binomial transformer Q distribution and the LEP Q-distribution expressions, a piecewise closed-form Q-distribution expression is developed, which is applicable over a wide range of total Q values.**

***Index Terms*—Bandpass filters, binomial transformer, Butterworth filters, maximally flat magnitude filters, quarter-wavelength-coupled filters, Q distribution.**

I. Introduction

To accurately design a maximally flat quarter-wavelength-coupled transmission-line filter using arbitrary resonant elements, one needs to know the distribution of Q values for the resonant sections of the filter [1]. Matching this Q distribution to the Q of the resonator sections, a maximally flat response is created. For a given number of resonant sections and total Q, there is a unique distribution of Q values for the individual resonators. Unfortunately, a closed-form solution for the Q distribution, which is valid for all values of total Q, does not exist.

This paper develops a closed-form expression for the Q distribution. This expression is derived from the binomial transformer, a closed-form equation that has been used for designing maximally flat quarter-wave transformer impedance-matching networks (IMN's) [2]–[5]. Specifically, the binomial transformer equation solves for the impedance of each section given the number of sections and the load-to-source mismatch. By substituting the binomial transformer equation into an expression for the Q of each quarter-wave transformer section, a closed-form expression for the Q distribution is created. This expression depends on the load-to-source mismatch. However, to be useful to a designer, the Q distribution needs to be expressed in terms of the total Q of the filter. Therefore, a relation is derived between the load-to-source mismatch of an IMN and the total Q and, from this relation, the resulting closed-form Q-distribution expression depends only on the total Q of the filter.

It is shown that the derived closed-form expression is only valid for low total Q filters. This is because the binomial transformer equation is only valid for small load-to-source mismatches, which implies that it is only valid for small values of total Q. Currently, there exists another approximate closed-form expression for the Q distribution, which is based on the lumped-element prototype (LEP) filter. It is also shown that the LEP Q-distribution expression is only valid for high total Q filters. By combining the LEP and binomial transformer Q-distribution expressions in a piecewise fashion, an overall Q-distribution expression is offered, which is accurate for most practical values of total Q.

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II. Deriving the Q Distribution from the Binomial Transformer

In this section, a closed-form expression of the Q distribution is derived by substituting the binomial transformer impedance value into the equation for the Q of a quarter-wavelength transformer. First, the binomial transformer and the Q of a quarter-wave transformer section are discussed. Then, the binomial transformer impedance is substituted into the expression for the Q of each quarter-wave transformer section. This yields a closed-form expression of the Q distribution as a function of the load/source mismatch. Finally, the load-to-source mismatch is related to the total Q, which yields an equation for the Q distribution in terms of the total Q of the filter.

A. The Binomial Transformer

The binomial transformer equation gives approximate characteristic impedance values for each section of an n-section maximally flat quarter-wave transformer IMN. The binomial transformer equation is [2]–[5]

$$Z_{0i} = Z_0 \left(\frac{Z_L}{Z_0}\right)^{(M_i/2^n)}, \qquad i = 1, 2, \cdots, n \tag{1}$$

where Z_L is the load impedance, Z_0 is the source impedance, and n is the number of sections. M_i is related to coefficients of the binomial expansion by

$$M_i = \sum_{k=1}^{i} C_k \tag{2}$$

where C_k is the binomial coefficient given by

$$C_k = \frac{n!}{(n-k+1)!(k-1)!}, \qquad k = 1, 2, \cdots, n. \tag{3}$$

For simplicity, (1) can be normalized to Z_0, giving

$$z_{0i} = Z_{0i}/Z_0 = R^{M_i/2^n}, \qquad i = 1, 2, \cdots, n \tag{4}$$

where $R = Z_L/Z_0$ is the load-to-source mismatch.

As an example, for a two-section $(n = 2)$ IMN, $M_1 = 1$ and $M_2 = 1 + 2 = 3$ using (2) and (3). From (4), the normalized characteristic impedances are $z_{01} = R^{1/4}$ and $z_{02} = R^{3/4}$. This is in agreement with the same example given in [5].

The binomial transformer equation (4) is derived by considering the reflections that occur to a wave traveling through the IMN at each junction between quarter-wave transformer sections. At each junction, part of the wave is transmitted, and the other part is reflected. When the reflected wave reaches the previous junction, a part is again reflected and a part is transmitted, and so on. Summing all of the reflections/transmissions at each junction gives an accurate mathematical description of the wave throughout the IMN. With this description, the maximally flat form can be found by setting the derivatives of the function, evaluated at the center frequency, to zero. Although the mathematical description is quite complex, by considering only the first-order reflections, the expression is significantly simplified. This simplified expression is the binomial transformer equation.

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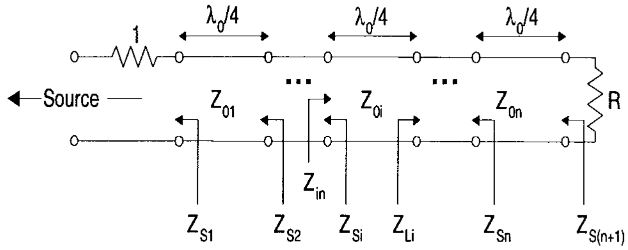

Fig. 1. A quarter-wavelength transformer IMN.

By assuming that multiple reflections do not occur, this implicitly assumes that the reflected wave is small in comparison to the transmitted wave. This occurs only when the difference between successive quarter-wave transformer impedances is small. Small impedance steps can occur only if the load/source mismatch R is small or, if R is large, there are many quarter-wave transformer sections, i.e., n is large. Thus, the binomial transformer approximation loses accuracy for large R and small n. We will show in Section II-E that as R gets large or n gets small, the total Q, Q_T of the IMN gets large. As a result, because large R and small n IMN's are high Q_T IMN's, the binomial transformer expression is only valid for low Q_T IMN's.

B. The Q for a Quarter-Wavelength Transformer

Fig. 1 shows a quarter-wavelength transformer IMN. Each section has a characteristic impedance Z_{0i}, a real source resistance Z_{Si}, and a real load resistance equal to Z_{Li}.

The Q for a single quarter-wavelength transformer is found by using [6], [4]

$$Q = \left[\frac{\omega}{2R_s}\,\frac{\partial X_s}{\partial \omega}\right]_{\omega=\omega_0} \tag{5}$$

where R_s is the total resistance and X_s is the total reactance in series. Substituting the expressions for R_s and X_s into (5) gives an expression for the loaded Q of a quarter-wavelength section of transmission line terminated in Z_{Li} as [6]

$$Q_i = \frac{\pi}{8}\left|\frac{Z_{0i}}{Z_{Li}} - \frac{Z_{Li}}{Z_{0i}}\right| \tag{6}$$

or, equivalently, as

$$Q_i = \frac{\pi}{8}\left|\frac{Z_{Si}}{Z_{0i}} - \frac{Z_{0i}}{Z_{Si}}\right| \tag{7}$$

by using the familiar quarter-wavelength transformer condition

$$Z_{Si} = \frac{Z_{0i}^2}{Z_{Li}}. \tag{8}$$

C. Q Distribution as a Function of R

1) Substituting Binomial Transformer Impedance into Q_i: Combining (4) and (6) yields an equation for Q for a single quarter-wave section as a function of the overall load/source mismatch R as follows:

$$Q_i = \frac{\pi}{8}\left|\frac{R^{M_i/2^n)}}{Z_{Si}} - \frac{Z_{Si}}{R^{M_i/2^n)}}\right|, \qquad i = 1, \cdots, n \tag{9}$$

where Z_{Si} is the junction impedance, i.e., the impedance looking toward the source. This junction impedance Z_{Si} can be specified in terms of the previous quarter-wave-section characteristic impedances by using (8) (with $Z_{S1} = Z_0$) as follows:

$$\begin{aligned} Z_{S2} &= \frac{Z_{01}^2}{Z_{S1}} = \frac{Z_{01}^2}{Z_0} \\ Z_{S3} &= \frac{Z_{02}^2}{Z_{S2}} = \frac{Z_{02}^2 Z_0}{Z_{01}^2} \\ Z_{S4} &= \frac{Z_{03}^2}{Z_{S3}} = \frac{Z_{03}^2 Z_{01}^2}{Z_{02}^2 Z_0} \cdots . \end{aligned} \tag{10}$$

Generalizing, for i even,

$$Z_{Si} = \frac{Z_{0(i-1)}^2 Z_{0(i-3)}^2 \cdots Z_{01}^2}{Z_{0(i-2)}^2 Z_{0(i-4)}^2 \cdots Z_{02}^2 Z_0} \tag{11}$$

and i odd,

$$Z_{Si} = \frac{Z_{0(i-1)}^2 Z_{0(i-3)}^2 \cdots Z_{02}^2 Z_0}{Z_{0(i-2)}^2 Z_{0(i-4)}^2 \cdots Z_{01}^2}. \tag{12}$$

2) Q_i for i Even: By substituting (1) into (11), Z_{Si} is expressed in terms of R as

$$Z_{Si} = \frac{R^{2M_{i-1}/2^n} R^{2M_{i-3}/2^n} \cdots R^{2M_1/2^n}}{R^{2M_{i-2}/2^n} R^{2M_{i-4}/2^n} \cdots R^{2M_2/2^n}}. \tag{13}$$

Substituting (13) into (9) yields the following expression:

$$Q_i = \frac{\pi}{8}\left|\frac{R^{M_i/2^n}}{\dfrac{R^{2M_{i-1}/2^n} R^{2M_{i-3}/2^n} \cdots R^{2M_1/2^n}}{R^{2M_{i-2}/2^n} R^{2M_{i-4}/2^n} \cdots R^{2M_2/2^n}}} - \frac{\dfrac{R^{2M_{i-1}/2^n} R^{2M_{i-3}/2^n} \cdots R^{2M_1/2^n}}{R^{2M_{i-2}/2^n} R^{2M_{i-4}/2^n} \cdots R^{2M_2/2^n}}}{R^{M_i/2^n}}\right| \tag{14}$$

which may also be written as

$$Q_i = \frac{\pi}{8}\left|R^{1/2n(M_i - 2M_{i-1} + 2M_{i-2} - \cdots + 2M_2 - 2M_1)} - R^{-1/2n(M_i - 2M_{i-1} + 2M_{i-2} - \cdots + 2M_2 - 2M_1)}\right|. \tag{15}$$

Finally, substituting (2) into (15) yields

$$Q_i = \frac{\pi}{8}\left|R^{1/2n(C_i - C_{i-1} + C_{i-2} - \cdots - C_3 + C_2 - C_1)} - R^{-1/2n(C_i - C_{i-1} + C_{i-2} - \cdots - C_3 + C_2 - C_1)}\right|. \tag{16}$$

3) Q_i for i Odd: Similarly, for i odd, by substituting (1) into (12), Z_{Si} is expressed in terms of R as

$$Z_{Si} = \frac{R^{2M_{i-1}/2^n} R^{2M_{i-3}/2^n} \cdots R^{2M_2/2^n}}{R^{2M_{i-2}/2^n} R^{2M_{i-4}/2^n} \cdots R^{2M_1/2^n}}. \tag{17}$$

Substituting (17) into (9) and simplifying using (2) yields

$$Q_i = \frac{\pi}{8}\left|R^{1/2n(C_i - C_{i-1} + C_{i-2} - \cdots - C_3 + C_2 - C_1)} - R^{-1/2n(C_i - C_{i-1} + C_{i-2} - \cdots + C_3 - C_2 + C_1)}\right|. \tag{18}$$

D. General Expression for Q Distribution as a Function of R

Noting that Q_i in (16) with i even and Q_i in (18) with i odd are identical in form, the general expression for the Q distribution as a function of R is

$$Q_i = \frac{\pi}{8}\left|R^{N_i/2n} - R^{-N_i/2n}\right|, \qquad i = 1, \cdots, n \tag{19}$$

where

$$N_i = \sum_{k=1}^{i} (-1)^{i-k} C_k. \tag{20}$$

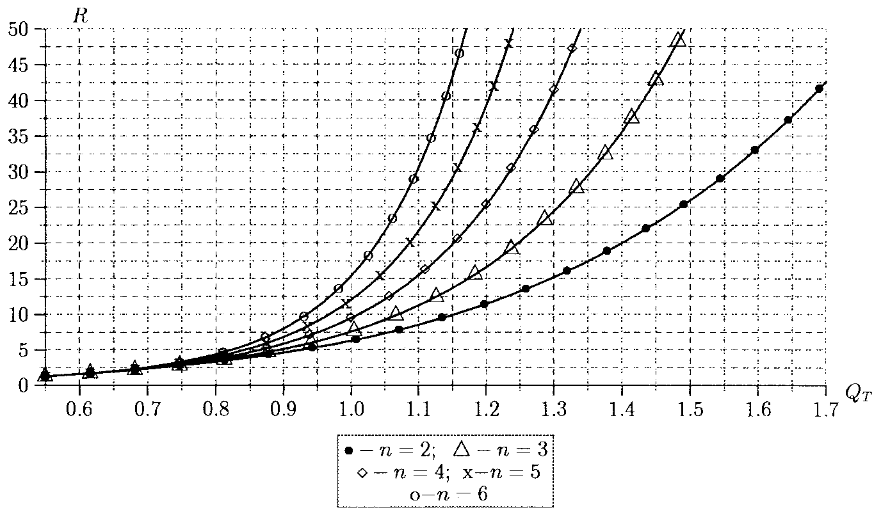

Fig. 2. R versus Q_T using different numbers of sections (n).

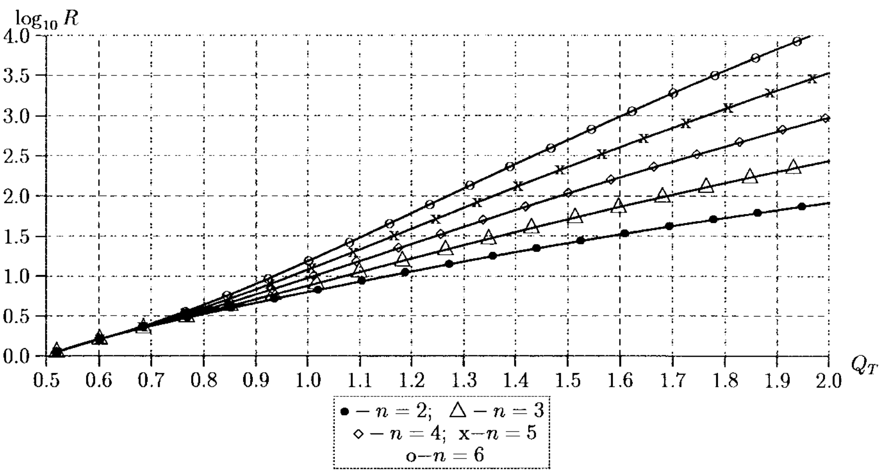

Fig. 3. $\log_{10} R$ versus Q_T using different numbers of sections (n).

The values for N_i can also be obtained from the following modified version of Pascal's triangle:

$$\begin{array}{lccccccccc} n=1 & & & & & 1 & & & & \\ n=2 & & & & -1 & & 1 & & & \\ n=3 & & & 1 & & -2 & & 1 & & \\ n=4 & & -1 & & 3 & & -3 & & 1 & \\ n=5 & 1 & & -4 & & 6 & & -4 & & 1. \end{array} \quad (21)$$

E. R Versus Total Q

The Q distribution given by (9) is specified in terms of R, but the Q distribution for a given filter is typically specified as a function of Q_T. Thus, it is necessary to relate R to Q_T. While a closed-form relation between Q_T and R has not been found, Fig. 2 provides a graphical relation between R versus Q_T for several values of n. Fig. 2 shows that as R increases, Q_T increases, for a given number of sections n. Notice also, using more sections results in a lower Q_T for a given R.

An approximate relationship between R and Q_T is developed by noting that the graph of $\log_{10}\ R$ versus Q_T, shown in Fig. 3, is basically linear for low values of Q_T. Using this linearity, the following functional form is obtained:

$$10\log_{10} R = (4n+5)Q_T - 3n + 1 \quad (22)$$

which may also be written as

$$R = [10]^{(1/10)((4n+5)Q_T - 3n+1)}. \quad (23)$$

F. Binomial Transformer Q-Distribution Approximation

Substituting (23) (the approximate relationship between R and Q_T) into (19) (the binomial transformer Q distribution) yields an expression for the Q distribution as a function of Q_T and n as follows:

$$Q_i = \frac{\pi}{8}\left|10^{(1/10)(N_i/2^n)((4n+5)Q_T-3n+1)} - 10^{(-1/10)(N_i/2^n)((4n+5)Q_T-3n+1)}\right|, \qquad i = 1, \cdots, n. \quad (24)$$

As an example, the Q distribution for a four-section filter with $Q_T = 0.8$ is calculated as follows. Using Fig. 2, $Q_T = 0.8$ corresponds to $R = 3.751$. Using the closed-form approximation (23), $Q_T = 0.8$ corresponds to $R = 3.802$. With this information, the binomial transformer Q distribution is found from (9) or (24). The resulting Q distributions and the actual Q distribution are given in Table I. Note that the values of Q_i obtained using the binomial transformer expression are fairly close to the actual Q distribution. Also notice that the Q distribution created using the binomial transformer is symmetrical, i.e., $Q_1 = Q_4$ and $Q_2 = Q_3$.

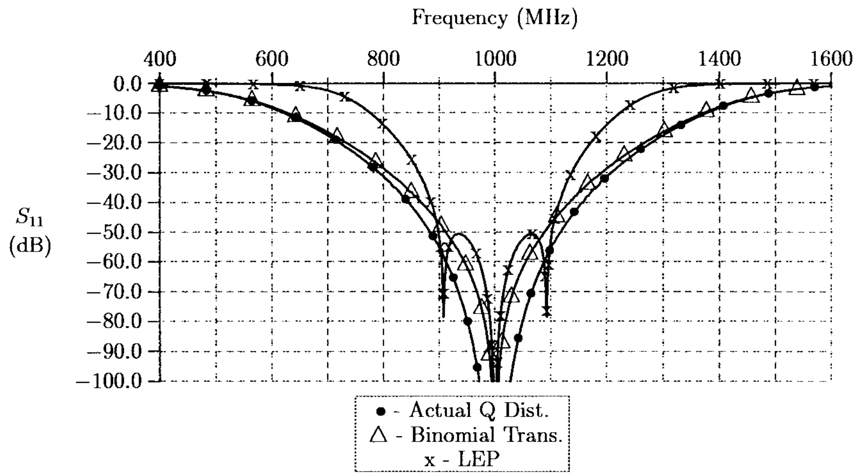

Fig. 4. Comparison of results for a four-section filter with $Q_T = 1.0$.

TABLE I
BINOMIAL TRANSFORMER Q DISTRIBUTION FOR A FOUR-SECTION FILTER, $Q_T = 0.8$

Parameter	Bin. QD Exact R	Bin. QD App. R	Actual Q Dist.
R	3.751	3.802	
Z_{S1}	1.000	1.000	
Z_{01}	1.086	1.087	
Z_{S2}	1.180	1.182	
Z_{02}	1.512	1.518	
Z_{S3}	1.937	1.950	
Z_{03}	2.482	2.505	
Z_{S4}	3.180	3.217	
Z_{04}	3.453	3.497	
Q_1	0.065	0.066	0.067
Q_2	0.197	0.199	0.194
Q_3	0.197	0.199	0.194
Q_4	0.065	0.066	0.067

TABLE II
BINOMIAL TRANSFORMER VERSUS ACTUAL Q DISTRIBUTION FOR A THREE-SECTION FILTER

Q_T	Actual Q_1	Q_2	Binomial Q_1	Q_2	R
0.60	0.047	0.095	0.047	0.095	1.620
1.00	0.205	0.409	0.202	0.416	7.628
1.50	0.424	0.849	0.403	0.906	51.57
2.00	0.658	1.316	0.599	1.506	277.3
3.00	1.141	2.283	0.962	3.040	3832.
5.00	2.128	4.256	1.598	7.245	117214.
10.00	4.618	9.235	2.926	22.567	1.092×10^7

III. A PIECEWISE EXPRESSION FOR THE Q DISTRIBUTION

The binomial transformer Q distribution is only accurate for small load/source mismatch R (low Q_T), and/or a large number of sections n. Another Q-distribution approximation that currently exists is based on the LEP circuit and is valid for high Q_T filters. Using a piecewise combination of both the LEP and binomial transformer Q-distribution expressions, an overall closed-form relationship can be developed, which is fairly accurate over a wide range of Q_T values.

A. Valid Range of the Binomial Transformer Q Distribution

Tables II and III, compare the results for the binomial transformer Q distribution against the actual Q distribution for $n = 3$ and $n = 6$. Notice that for $n = 3$ and $n = 6$, the binomial transformer Q

TABLE III
BINOMIAL TRANSFORMER VERSUS ACTUAL Q DISTRIBUTION FOR A SIX-SECTION FILTER

Q_T	Actual Q_1	Q_2	Q_3	Binomial Q_1	Q_2	Q_3	R
0.60	0.006	0.030	0.058	0.006	0.029	0.060	1.616
1.00	0.044	0.181	0.323	0.034	0.169	0.346	15.47
1.50	0.124	0.453	0.751	0.076	0.397	0.889	502.6
2.00	0.224	0.763	1.209	0.116	0.633	1.626	12538.
3.00	0.451	1.426	2.153	0.180	1.098	3.776	2.091×10^6
5.00	0.943	2.804	4.068	0.267	2.015	11.095	1.952×10^9
10.00	2.216	6.313	8.886	0.391	4.270	47.202	2.049×10^{13}

distributions are more accurate for low values of Q_T than for high values of Q_T. Also, for a given value of Q_T, the binomial transformer Q distribution for a filter with fewer sections is more accurate. In fact, the binomial transformer Q distributions for $n = 2$ are identical to the actual Q distributions for all values of Q_T.

B. LEP Q Distribution

The LEP Q-distribution expression begins with the element values ($g_k = L_k$ or C_k) for a maximally flat response [7]

$$g_k = 2 \sin\left[\frac{(2k-1)\pi}{2n}\right], \qquad k = 1, 2, \cdots, n. \tag{25}$$

Using this expression, the Q distribution of each individual resonator k is given by [4]

$$Q_k = Q_T \sin\left[\frac{(2k-1)\pi}{2n}\right], \qquad k = 1, 2, \cdots, n. \tag{26}$$

Unfortunately, (26) does not result in a maximally flat response and does not give the correct total Q [1]. The reason is that the quarter-wavelength sections of transmission line contribute to the filter response. For high total Q filters, this effect is not as noticeable, but for low total Q filters, this error causes ripples in the passband. In addition, the transmission-line selectivity adds to the Q of the filter. Thus, using (26) results in an inaccurate Q_T. It approaches the correct value only at high values of Q_T.

C. Low and High Total Q Example Comparison

To compare the valid range of the LEP Q distribution to the binomial transformer Q distribution, a low total Q example, $Q_T = 1.0$, and a high total Q example, $Q_T = 5.0$, are created. Both examples are for a four-section quarter-wavelength-coupled filter

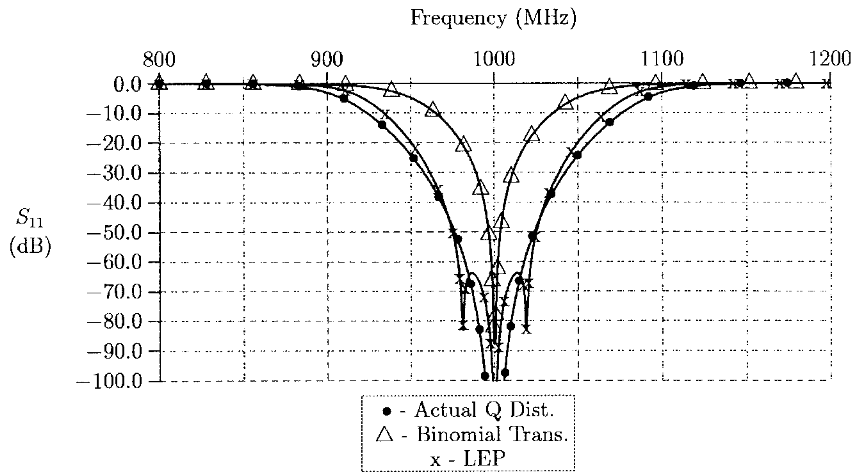

Fig. 5. Comparison of results for a four-section filter with $Q_T = 5.0$.

TABLE IV
BINOMIAL TRANSFORMER VERSUS LEP Q DISTRIBUTION FOR A FOUR-SECTION FILTER WITH $Q_T = 1.0$

Parameter	Q Dist.	Bin. Trans.	LEP
Q_1	0.119	0.111	0.383
Q_2	0.334	0.342	0.924
Z_{q1}	165.260 Ω	176.826 Ω	51.306 Ω
Z_{q2}	58.852 Ω	57.412 Ω	21.252 Ω
Q_{3dB}	1.000	1.005	1.733

TABLE V
BINOMIAL TRANSFORMER VERSUS LEP Q DISTRIBUTION FOR A FOUR-SECTION FILTER WITH $Q_T = 5.0$

Parameter	Q Dist.	Bin. Trans.	LEP
Q_1	1.549	0.843	1.914
Q_2	3.873	6.417	4.620
Z_{q1}	12.679 Ω	23.286 Ω	10.261 Ω
Z_{q2}	5.070 Ω	3.060 Ω	4.250 Ω
Q_{3dB}	5.000	8.562	5.814

that uses quarter-wavelength shorted-stub resonators, which have Q values given by

$$Q_i = \frac{\pi}{8} \frac{Z_0}{Z_{qi}} \tag{27}$$

where Z_{qi} is the impedance of the ith quarter-wavelength shorted stub and $Z_0 = 50\ \Omega$. In addition, both example filters are designed to resonate at $f_0 = 1$ GHz.

For the low total Q example, $Q_T = 1.0$, Fig. 4 shows S_{11} from modeled results for the exact Q distribution (using the Q-distribution method), with the LEP expression (26) and with the binomial transformer expression (9). Table IV provides a summary of the results. $Q_{3\text{ dB}}$ is calculated using the 3-dB bandwidth

$$Q_{3\text{ dB}} = \frac{f_0}{f_2 - f_1} \tag{28}$$

where f_2 and f_1 are upper and lower 3-dB frequencies, respectively. Notice that the LEP Q distribution yields large ripples in the passband, and does not accurately give the desired value for Q_T. On the other hand, the binomial transformer Q distribution is very accurate.

For the high total Q example, $Q_T = 5.0$, Fig. 5 shows S_{11} from modeled results using the actual Q values (the Q-distribution method), with the LEP expression for the Q distribution, and with the binomial transformer expression Q values. Table V summarizes the results. Notice that the LEP expression still creates ripples in the passband, but fairly accurately gives the correct value for Q_T. On the other hand, using the binomial transformer Q distribution gives inaccurate results, but does not have ripples in the passband.

D. Closed-Form Approximation for the Q Distribution

Since the binomial transformer Q distribution is fairly accurate for low Q_T filters, and the LEP Q distribution is fairly accurate for high Q_T filters, the following piecewise closed-form expression for the Q distribution is offered:

$$\begin{aligned} Q_i &= \frac{\pi}{8} \Big| 10^{(1/10)(N_i/2^n)((4n+5)Q_T - 3n+1)} \\ &\quad - 10^{(-1/10)(N_i/2^n)((4n+5)Q_T - 3n+1)} \Big|, \qquad Q_T \le 3.0 \\ Q_i &= Q_T \sin\left[\frac{(2i-1)\pi}{2n}\right], \qquad Q_T > 3.0 \quad i = 1, 2, \cdots, n. \end{aligned} \tag{29}$$

The value of $Q_T = 3.0$ was arbitrarily chosen as the cutoff. It may be more useful to define this cutoff as a function of the number of sections n.

IV. CONCLUSION

This paper has presented a derivation of a closed-form expression for the Q distribution based on the binomial transformer equation, which is often used to design maximally flat IMN's. It has been shown that this expression is accurate for low total Q filters. Since the current closed-form expression based on the LEP circuit is accurate for high total Q filters, a piecewise closed-form expression for the Q distribution was also offered, which is applicable to filters for all values of total Q.

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Circuit Theory for Spatially Distributed Microwave Circuits

Ahmed I. Khalil and Michael B. Steer

***Abstract*— A spatially distributed radio-frequency (RF) circuit, microwave, or millimeter-wave circuit does not have a global reference node as required in conventional nodal analysis. Instead, local reference nodes associated with ports are required. This paper adapts modified nodal analysis to accommodate spatially distributed circuits, allowing conventional harmonic balance and transient simulators to be used.**

***Index Terms*— Circuit simulation, computer-aided design, microwave circuits.**

I. Introduction

Nodal analysis is the mainstay of circuit simulation. The basis of the technique is relating nodal voltages (voltages at nodes referenced to a single common reference node) to the currents entering the nodes of a circuit. Generally, the art of modeling is then to develop a current/nodal-voltage approximation of the physical characteristics of a device or structure. With spatially distributed structures, a reasonable approximation can sometimes be difficult to achieve. The essence of the problem is that a global reference node cannot reasonably be defined for two spatially separated nodes when the electromagnetic field is transient or alternating. In this situation, the electric field is nonconservative and the voltage between any two points is dependent on the path of integration and, hence, voltage is undefined. This includes the situation of two separated points on an ideal conductor. In a time-domain context, it takes a finite time for the state at one of the points on the ideal conductor to affect the state at the other point. In the case of waveforms on digital interconnects, this phenomenon has become known as retardation [1]. With high-speed digital circuits, it is common to model ground planes by inductor networks so that interconnects are modeled by extensive meshes of resistors, inductors, and capacitors. Consequently, no two separated points are instantaneously coupled. In transient analysis of distributed microwave structures, lumped-circuit elements can be embedded in the mesh of a time discretized electromagnetic-field solver such as a finite-difference time-domain (FDTD) field modeler [2], [3]. The temporal separation of spatially distributed points is then inherent to the discretization of the mesh.

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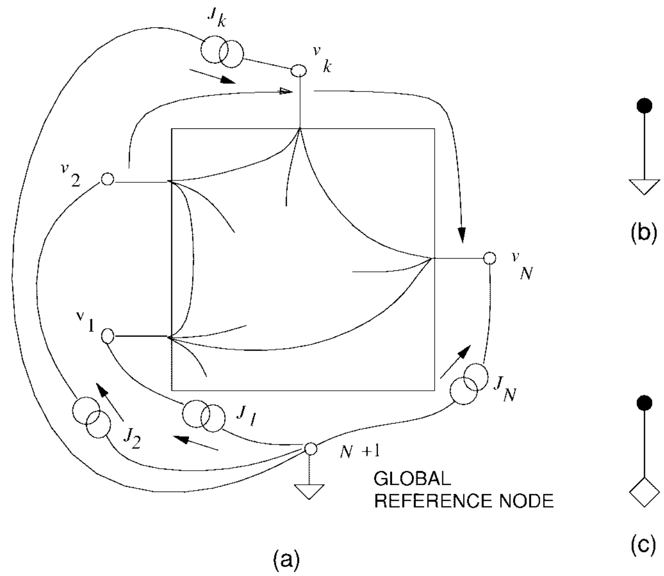

Fig. 1. Nodal circuits. (a) General nodal circuit definition. (b) Conventional global reference node. (c) Local reference node proposed here.

With a frequency-domain electromagnetic-field simulator, ports are defined and, thus, a port-based representation of the linear distributed circuit is produced. With ports, a global reference node is not required. Instead, a local reference node (one of the terminals of the two-terminal port) is implied. The beginnings of a circuit theory incorporating ports in circuit simulation has been described and termed the compression matrix approach [4], [5]. This milestone work presented a technology for integrating port-based electromagnetic-field models with nonlinear devices. Circuit simulation using port representation has been reported in [6]. This requires the representation of nodally defined circuits in its port equivalent by a general-purpose linear multiport routine. Hence, the advantage of accessing information at all nodes, as in nodal analysis, is lost.

The purpose of this paper is to extend the circuit theory behind the compression matrix approach to general-purpose circuit simulators based on nodal analysis. In particular, we present the concept of local reference nodes that enables port-based network characterization to be used with nodally defined circuits in the development, by inspection (the preferred approach), of what is termed a locally referenced nodal admittance matrix. A procedure for handling and moving the local reference nodes is described, along with circuit-reduction techniques that facilitate efficient simulation of nonlinear microwave circuits.

II. Nodal-Based Circuit Simulation

The most popular method for circuit analysis in the frequency domain is the nodal admittance matrix method. In the nodal formulation of the network equations, a matrix equation is developed that relates the unknown node voltages to the external currents using the model shown in Fig. 1. All node voltages are then defined with respect to an arbitrarily chosen node called the global reference node. Eliminating the row and column associated with the global reference node leads to a definite admittance matrix, and then the solution for the node voltages is straightforward. In this type of analysis, only one reference node can exist.

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